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Website: ijcs.stmikindonesia.ac.id | E-mail: ijcs@stmikindonesia.ac.id

Fluctuation Behavior of The Degree Growth Dynamics in Complex Networks

Han-Yun Chang [1], Jiang-Hai Qian[1*)]
qianjianghai@shiep.edu.cn, changhanyun@163.com
[1]College of Mathematics and Physics, Shanghai University of Electric Power

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Abstract

The standard network theory predicts the fluctuation of the degree growth rate is interval-independent, whereas the evidence witnessed in Internet shows an inconsistent result, which raises the concern of many existing relevant studies that apply just single observation interval. To check whether such inconsistency occurs in more systems, we study empirically the degree growth fluctuations in two social networks. We find both their fluctuation exponents decrease logarithmically with the observation interval, presenting clear interval dependency that differs from those observed in Internet in the specific manner but is still consistent in the tendency. By applying a progressive shuffling procedure, we find an asynchronous response of the fluctuation exponent and deduce the decline of the exponent might be related to the development of the internal correlation. These results indicate the general existence of the interval dependency in the degree growth fluctuation and suggest its close connection with the correlation evolution, which could provide new insight to the related dynamics in complex networks.

A. Introduction

The evolutions of many self-organized systems across a number of fields such as socioeconomics, ecology and biology are governed by the proportionate effect[1–4], whose mathematical description is formalized by the random multiplicative process

$$k(t + \Delta t) = R(t, \Delta t)k(t), \qquad (1)$$

where Δt is the observation interval, k(t) and k(t + Δt) represent certain quantity at the corresponding time and R(t, Δt) is a random process, whose logarithmic value r = lnR is called the growth rate. The fluctuation of the growth rate $\sigma_r(k)$ is defined as the standard deviation of r(Δt) conditional to the initial quantity k(we omit t for brevity here as well as in the rest of the paper), which generally scales as

$$\sigma_r(k) \sim k^{-\alpha}, \qquad (2)$$

where α is termed as the fluctuation exponent. In its primary version, which is also known as Gibrat's law[5], thegrowth rate of unit interval is assumed to be independent identically distributed. According to the central-limit theorem, one can show $\sigma_r(k)$ ∼const. On the other hand, the standard network theory demonstrates the fluctuation of the degree growth rate under the rule of preferential attachment follows $\sigma_r(k)$ ∼ k−0.5[6–9]. We therefore have two theoretical regimes characterized by the fluctuation exponents α = 0 and α = 0.5 respectively, and both are independent of the observation interval Δt.

However the fluctuations in the real-world systems belong to neither of the regime, but instead lie somewhere in between. H. E. Stanely et al. found in firm growth[10]that the fluctuation exponent was about 0.15. They pointed out that the primary Gibrat's law with α = 0 corresponded to a completely correlated state between divisions in firm, while at the other end α = 0.5 was the uncorrelated situation. Thus α with value in between in dicated a particular correlation strength. Similar results were obtained in numbers of the later empirical studies including population growth, economic volatility and human dynamics[6, 11–15]. D.Rybski studied the fluctuation in human communication behavior[6]. They found the fluctuation exponent in the growth of the received message was close to that observed in firm growth. By applying a standard detrended fluctuation analysis, they confirmed the intermediate exponent is related to the long-term correlation in the human behavior and derive a proportional relation between the fluctuation exponent and the decay rate of the correlation function[6, 14, 15].With evidences accumulated, the growth-rate fluctuation has been considered as an effective means to analyze the underlying correlation and the modified models were proposed to produce consistent α[16–21]. Nonetheless, all these studies focus on the fluctuation behavior of only single interval Δt, which indicates somewhat tacit consent to the interval-independent assumption as predicted by both the primary Gibrat's law and the standard network theory. But in fact this assumption seems far from fully validated.

We have applied the fluctuation analysis to the degree growth dynamics in Internet[22–24] and found a new fluctuation pattern that both α = 0 and α = 0.5 phase coexist, with a critical degree kc separating them. What drew our attention was that when we tuned the observation interval Δt, kc changed simultaneously, leading to a crossover from BA type to the primary Gibrat's law. This interval-dependent pattern is inconsistent with the traditional assumption and raises the concern that whether the fluctuation exponents obtained previously will also vary with the observation interval[16–21]. If this is true,taking the fluctuation exponent of a single interval as the characterization of the correlation strength seems quitearbitrary. It also raises the doubt of the validity of those models that generate the fixed fluctuation exponent. As the issue concerns numbers of the existing studies, it is necessary to check its universality. Therefore in this paper, we study the degree growth fluctuation of two social networks and present more evidences for the existence of the interval-dependent pattern. The paper is organized as follows. In section II, we show the details of our data source, network construction and methodology of the statistical analysis. In section III, we present the empirical results and discuss the possible origin of the interval- dependent fluctuations, and in section IV, we draw the conclusion.

B. Materials and Methods

Our empirical data include two complex networks. The first one is the Bitcoin trust network(BTN)[25, 26], which describes who-trusts-whom relation of people who trade using Bitcoin on a platform called Bitcoin OTC. Since Bitcoin users are anonymous, there is a need to maintain a record of users' reputation to prevent transactions with fraudulent and risky users. Members of Bitcoin OTC rate other members in a scale of −10 to +10 and an edge is created when this activity occurs between two users, which forms a network with total 5881 nodes and 35592 edges. The other one is the chess network(CN)[27] where nodes are chess players and edges represent games between them. The network has 7301 nodes and 65053 edges in total. The time span of the data is about 60 months for BTN and 96 months for CN. Note that both data sources include detailed timestamp for every activity, so that we can trace the temporal order and reconstruct their dynamic evolution. The minimum time resolution to reconstruct the evolution is set to 6 months and 12 months respectively. The time for constructing the initial network is selected properly, resulting in 691 nodes, 2184 edges for BTN and 1162 nodes, 3536 edges for CN. These settings guarantee the sufficient data volume for statistical analysis in every Δt as well as the widevalue range of Δt for $\alpha(\Delta t)$ presentation.

Our empirical study focuses on the degree growth rate

$$r_i(t) = \ln \frac{k_i(t+\Delta t)}{k_i(t)} , \qquad (3)$$

where ki is the degree of node i. The fluctuation is described by $\sigma_r(k)$, which is defined as

$$\sigma_r(k_i) = <\sqrt{<r_i^2> - <r_i>^2} \quad , \qquad (4)$$

where <·> represents the average over the resemble with the same ki. In practice, we consider all ri within [ln ki − Δl, ln ki + Δl] as the resemble with ki, i.e. the logarithmic bin technique is used to reduce the noise in calculating the conditional standard deviation. For a particular Δt, we move the time window to measure the fluctuation exponent αt for all possible t and use their average α as the representative. We do this for different Δt and then investigate the relation between α and Δt.

C. Results and Discussion

Fig. 1(a) and (b) show the empirical results of the degree growth fluctuations of BTN and CN. Their $\sigma_r(k)$ display power-law decrease as described by Eq(2) for all intervals. Remarkably, the fitting lines in both figures become gradually flat with the increasing Δt, presenting a clear interval-dependent picture. To characterize quantitatively the pattern of the dependence, we study the relation between the average fluctuation exponent α and the interval Δt. As shown in Fig 2, the α(Δt) in both networks exhibit logarithmic decrease

$$\alpha(\Delta t) \sim \mu \ln \Delta t, \qquad (5)$$

where the slope μ is measured as 0.025 and 0.167 for BTN and CN respectively. The patterns are different from our previous findings in Internet[22–24]. Nonetheless, we notice they all display a tendency towards the primary Gibrat's law(α = 0) with increasing Δt, which implies possible universal law behind the diverse crossover patterns.

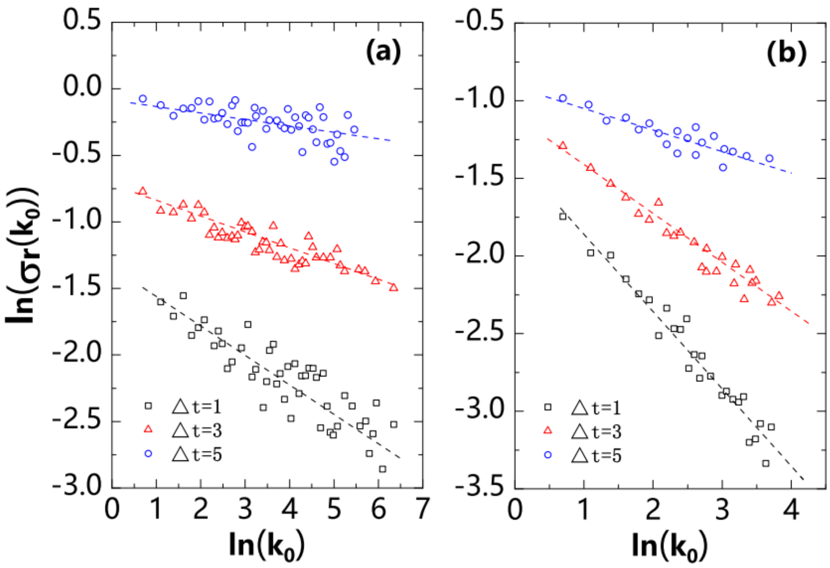

Figure. 1: The degree-growth fluctuations for (a)BTN and (b)CN. The $\sigma_r(k)$ of both networks follow power-law decay while their slopes in log-log plot decrease with the interval Δt. The blue and black data points in both figures are moved up or down in parallel for better visualization. The dashed lines are the fitting lines used as guides to eyes.

According to the modern Gibrat-law theory[6, 10], the fluctuation exponent is inversely related to the correlation strength. The observed decline of α(Δt) then, by inference, is a manifestation of the reinforcement of the internal correlation, which presents a dynamical evolution picture. To capture a sign associated with such dynamics, we apply a shuffling procedure to remove the correlation from the original data and study the response of the degree growth fluctuation. In specific, we choose p percentage of the links and exchange their temporal position both in a random way. If p = 100%, all the correlations are destroyed and the fluctuation exponents α are expected to be 0.5 regardless of the interval Δt. Nonetheless, during the process of progressively increasing p, the response of α(Δt) can be nontrivial and provide insight into the underlying correlation behavior.

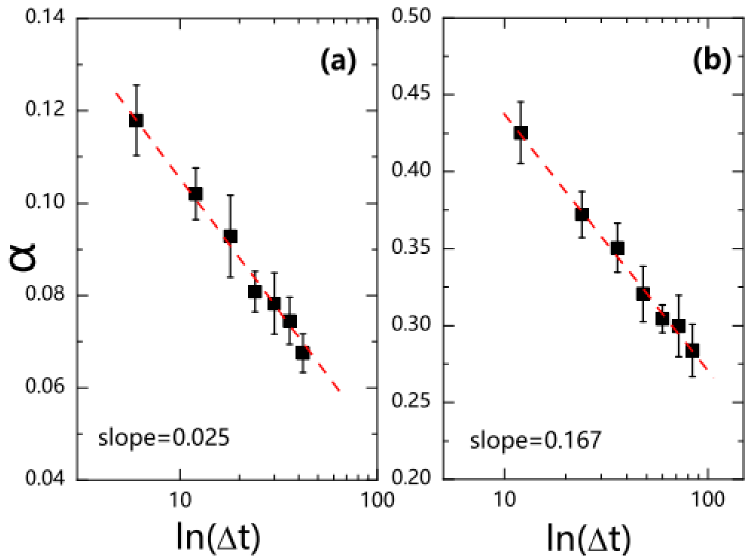

Figure. 2: The relation between the average fluctuation exponent α and the interval Δt for (a)BTN and (b)CN. They both follow logarithmic decrease at the rate of 0.025 and 0.167 respectively, as measured by the red dashed fitting lines.

In Fig. 3(a) and (b) we present the results of p = 20%(black square), p = 50%(green triangle) and p = 100%(blue circle) for BTN and CN respectively. As expected, when p = 100%, the fluctuation exponents of both networks become independent of the interval and all reach 0.5. However for other p, the α shows an apparentΔt dependent performance in response to the randomization. In BTC, when p = 20%, the first two α begin to rise while the rest of them maintains almost original decrease. This asynchronous pattern is further amplified when p = 50%, where the rise of α with smaller Δt is more significant than those with larger ones, presenting a positive relation between the magnitude of the rise of α and their original value. The similar results are also observed in CN. The rise of the α of larger Δt shows an apparent lag and a lower magnitude compared to those of small Δt. The only exceptions are the first two data points of p = 50%, where the rise magnitude slow down. This consequence originates from the saturation effect. Since both the α get close to their upper bound, the majority of the correlation have been removed so that the further randomization will not cause any significant change in α.

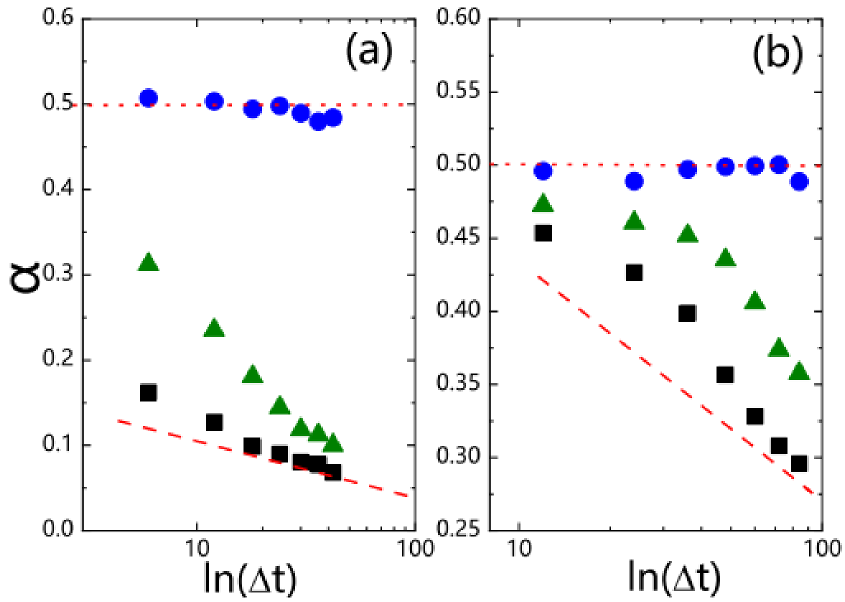

Figure. 3: Fluctuation exponents α versus the interval Δt for shuffled data: (a) BTN and (b) CN. The dashed lines represent the α(Δt) of the empirical data and the dotted horizontal lines represent the constant of 0.5. The black squares, green triangles and the blue circles correspond to the results of p = 20%, p = 50% and p=100% respectively. The exponents of all intervals reach near 0.5 and be-come independent of Δt after full shuffling. For p = 20%, p = 50%,we observe a more significant rise in the small Δt region, which presents an asynchronous response to the shuffling procedure.

To understand how this asynchronous response would happen, recall that a larger α corresponds to a correlation function with a higher decay rate[6, 14, 15] as indicated in our introduction, the shuffling procedure for larger α then is more likely to move the connections out of its correlation length and destroy the temporal correlation. If the decline of α in Fig. 2 is really related to the development of the correlation, α of larger Δt is expected to be more tolerant to the randomization. This explains the observed asynchronous response pattern and the heterogenous rise magnitude in α at some proper p.In other words, our shuffling experiment demonstrates the varying α is indeed related to the evolving correlation, which supports our inference on the origin of the interval-dependent fluctuation. As a consequence, the correlation is better to be characterized by the transition of α instead of any of its specific value.

D. Conclusions

We study the fluctuation behavior of the degree growth rate in two social networks and find both their fluctuation exponents decrease logarithmically with the observation interval, which shows a clear interval-dependent pattern.By applying a progressive shuffling procedure to the empirical data, we find an asynchronous rise in the response of the fluctuation exponents and deduce the interval-dependent pattern may result from the development of the temporal correlation. These results demonstrate the interval dependency is not just a fortuity but might commonly exist in many systems, and reveal its close connection with not only the correlation strength but also their evolution dynamics, which cannot be captured by those studies with just a single interval. Tough a firm conclusion needs further evidences, our study shows the necessity of the concentration on the interval-dependent behavior of the growth rate fluctuation and indicates its significance to disclosing the underlying mechanism of the correlation in complex systems.

E. References

[1] Mitzenmacher M 2004 Internet Mathematics 1 226
[2] Black F, Scholes M 1973 Journal of Political Economics 81 637
[3] Allen A P , Li B, Charnov E L 2001 Ecology Letters 4 1
[4] Jain R, Ramakumar S 1999 Physica A 273 476
[5] Gibrat R 1931 Les in´egalit´es ´economiques Recueil Sirey
[6] Rybski D et.al. 2009 Proc. Natl. Acad. Sci. USA 106 12640
[7] Barab´asi A L, Albert R 1999 Science 286 509
[8] Dorogovtsev S N, Mendes J F F, Samukhin A N 2000 Phys. Rev. Lett. 85 4633
[9] Qian J H, Zhao S T, Xu J 2021 Physica A 562 125333
[10] Stanely M H R, Amaral L A N, Buldyrev S V, Havlin S, Leschhorn H, Maass P, Sallnger M A, Stanely H E 1996 Nature 379 804
[11] Rozenfeld H D, Rybski D, Andrade J S, Batty M, Stanely H E, Makse H A 2008 Proc. Natl. Acad. Sci. USA 105 18702
[12] Plerou V, Amaral L A N, Gopikrishnan P, Meyer M, Stanely H E 1999 Nature 400 433
[13] Lee Y, Amaral L A N, Canning D, Meyer M, Stanely H E 1998 Phys. Rev. Lett. 81 3275
[14] Rybski D et.al. 2012 Scientific Reports 2 560
[15] Rybski D, Buldyrev S V, Havlin S, Liljeros F, Makse H A 2011 Eur. Phys. J. B 84 147
[16] Riccaboni M, Pammolli F, Buldyrevd S V, Pontac L, Stanely H E 2008 Proc. Natl. Acad. Sci. USA 105 19599
[17] Amaral L A N, Buldyrev S V, Havlin S, Salinger M A, Stanely H E 1998 Phys. Rev. Lett. 80 1385
[18] Fu D, Pammolli F, Buldyrev S V, Riccaboni M, Matia K, Yamasaki K, Stanely H E 2005 Proc. Natl. Acad. Sci. USA 102 18801
[19] Huberman B A, Adamic L A 1999 Nature 401 131
[20] Gautreau A, Barrat A, Barth´elemy M 2009 Proc. Natl. Acad. Sci. USA 106 8847
[21] Goh K I, Kahng B, Kim D 2002 Phys. Rev. Lett. 88 108701
[22] Qian J H, Chen Q, Han D D, Ma Y G, Shen W Q 2014 Phys. Rev. E 89 062808
[23] Chen Q, Qian J H, Han D D 2014 International Journal of Modern Physics C 25 1440012
[24] Qian J H 2018 Physica A 499 407
[25] Kumar S, Spezzano F, Subrahmanian V S, Faloutsos C 2016 IEEE International Conference on Data Mining 221-230
[26] Kumar S, Hooi B, Makhija D, Kumar M, Subrahmanian V S, Faloutsos C 2018 11th ACM International Conference on Web Searchand Data Mining 333-341
[27] Kunegis J 2013 In Proceedings of the 22nd international conference on world wide web 1343–1350