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Regulation of Network Condensation Based on Fitness Ordered Access Strategy

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Abstract

The condensation in complex networks disclose the underlying mechanism of the monopoly in socioeconomic system, which can help us to design and access the anti-monopoly policy based on the study on network condensation. Inspired by the consideration, We introduce a set of rearrangement mechanism into the fitness model to regulate the order of nodes with different fitness to enter the network, and study the influence of this regulation strategy on network condensation. By extensive Monte Carlo simulations and finite size scaling analysis, we obtain the critical rearrangement index under a typical fitness distribution, establish the relationship between the index and the condensation intensity, and finally construct the condensation phase diagram. These results show that there exists an interval of the critical rearrangement index, outside which the condensation of the fitness model will be effectively suppressed. We carry out a theoretical analysis on some key results to understand their underlying origin, and discuss their instructive significance on the anti-monopoly market management.

A. Introduction

Complex networks offer an elegant and universal theoretical framework for various socio-economic systems [1]. Extensive research on network evolution models has become an important paradigm for understanding the self-organizing principles of these systems [2-4]. Among these models, the fitness model [5], as one of the well-known models, simulates the dynamic process of nodes acquiring edges based on their intrinsic competitiveness(also termed as fitness) and successfully reproduces many empirical observations such as the "rich get richer" phenomenon and power-law degree distributions. Remarkably, the model predicts that when the fitness distribution satisfies some particular condition, the network will sprouts one or several super-hub nodes, who possess overwhelming competitive advantages over other ones and can acquire a considerable proportion of edges of the entire network.

Unlike the conventional high-degree nodes, these super hubs are usually separated from the main body of the distribution and are located on the extreme right side in the diagram[6]. From the dynamics perspective, the degrees of these nodes are supposed to grow proportionally with the total number of edges in the network, so that their edge share will not vanish in the thermodynamic limit. From the topology perspective, these nodes play the role of cores and result in the network of a star-like structure(Figure 1.). If the two ends of an edge are viewed as a pair of particles, and the fitness of the nodes at the ends of the edge is mapped to energy levels, the network system then can be analogized to a Bose gas. Based on this idea, G. Bianconi and A. L. Barabási discovered a mathematical correspondence between the emergence of these super-hub nodes and Bose-Einstein condensation, and thus named it network condensation [7]. In socio-economic systems, similar phenomena are also referred to "winner-takes-all" or "dragon king," which is a hot issue of interdisciplinary interest [8-10].

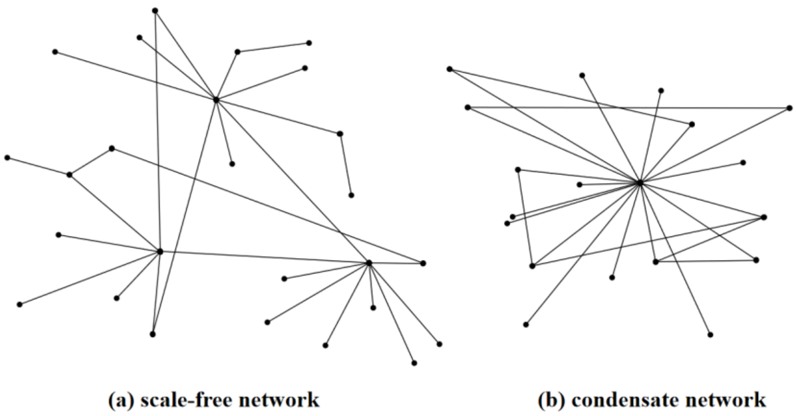

Figure 1. Comparison of the topology between scale-free and condensate network.

Since the discovery of the network condensation, scientists have conducted amounts of researches on its emergence condition, phase transition, and the related dynamics. L. Ferretti and G. Bianconi studied the dynamics of the condensation in fitness models and identified a deceleration characteristic during the emergence of condensed nodes [11]. They further generalized the fitness to negative situation and discovered a condensation induced by the edge rewiring [12]. O. Sotolongo-Costa and G. J. Rodgers studied the condensate transitions of both out-degree and in-degree in directed networks and identified a multi-phase coexistence state [13]. L. Ferretti and M. Cortelezzi found the condensation at metric singularities in spatial networks, where the fitness was expressed as a function of the spatial distance [14]. Other related studies include the Bose-Einstein condensation phenomenon in hypernetworks, cluster condensation in non-growing cooperative networks, aging or location selection effect on the condensate transition[15-19].In addition to condensation based on fitness models, some studies have explored other mechanisms. P. L. Krapivsky et al. proposed a growing random network model and found that condensed nodes emerge when the edge probability follows a superlinear relationship with degree [20]. Su Guifeng et al. combined this mechanism with fitness models and studied the parameter conditions for the condensate transitions[21-22]. V. Nicosia et al. introduced a similar superlinear preferential attachment in multilayer networks and induced the condensation within each layer of the network [23]. K. Anand et al. discovered that for a given degree distribution in configuration networks, the system exhibits condensation when the average degree exceeds a certain threshold. Note that this condensation is distinct from the Bose-Einstein condensation mechanism induced by fitness, as it only applies to certain large-deviation configurations [24].

Despite the various mechanisms and models, the original fitness model [5] is the most popular one due to its validity in characterizing competitive evolution in complex systems. In fact, if we consider the edges as the resources that the nodes acquire, the emerging condensation in the fitness model corresponds to an important phenomenon in competitive markets – the monopoly. Traditional anti-monopoly strategies mainly restrict the expansion and competitiveness of monopolies through taxation or legal enforcement. However, the fitness model indicates that the occurrence of a monopoly originates from the fitness environment in the system, rather than being caused by a specific individual. Inspired by this idea, Lera et al. developed a monopolistic alert system and proposed a fitness adjustment strategy based on the shortest cost path to counteract the condensation [25]. However, altering the fitness of the entire network requires huge resource investment, which motivates us to find a more applicable strategy. In this paper, we propose a regulation strategy of the network condensation by adjusting the entry order of the nodes with different fitness. The rest of the paper is organized as follow. In section II, we review the mathematical principle of the condensation in fitness model and discuss the theoretical basis of our ordered entry regulation; In section III, we propose the fitness reordering scheme for simulating the regulation and introduces a method for determining the condensate state; In section IV, we present the detailed results and discusses the practical significance of these results. In section V we come to the conclusion.

B. Theoretical basis of ordered admission strategy

The evolution mechanism of the fitness model [5] can be described as follows: at each moment, a node with attached edges is added to the network. Each of its edges independently connects independently to an existing node with probability , where represents the degree of node and represents the invariant fitness assigned to node when added to the network, which is drawn from a given distribution with . One can derive the dynamic equation of as

$$\frac{dk_i}{dt} = m\frac{\eta_i k_i}{\sum_j \eta_j k_j} \quad (1)$$

Noticing that is constrained between two linear relationships, namely , one can assume

$$\sum_j \eta_j k_j \sim Cmt \quad (2)$$

Substituting Eq(2) into Eq (1), we find the evolution of the degree . Substituting this solution back into Eq (2) and using the mean-field approach , we arrive at

$$\left\langle \sum_j \eta_j k_j \right\rangle = \int d\eta \frac{Cm\eta\rho(\eta)}{C-\eta} t(1-t^{\frac{\eta}{C}-1}) \quad (3)$$

If the condensation does not occur, for all (or equivalently)is required. The term in Eq (3) then vanishes in the thermodynamic limit. By combing Eq (3) with the Eq (2), we get a self-consistent equation for

$$\int_{\eta_{\min}}^{\eta_{\max}} d\eta\rho(\eta)\frac{\eta}{C-\eta} = 1 \quad (4)$$

Let represent the left side of Eq (4). Noticting that is a monotonically decreasing function with and , if , the continuity of the function ensures that there exists a solution satisfying Eq (4) when . We then can solve C from Eq(4) and determine a self-consistent non-condensate solution of . Conversely if , Eq (4) has no solution, indicating that the assumption of does not always hold for this . As a consequence there must exist an such that (note that > 1 is physically impossible), which implies the emergence of the network condensation.

The derivation of Eq (4), as the key criterion for the condensation, relies on the mean-field approach applied in Eq (3). The approach is valid when the addition of fitness is independent of the network evolution. However, if there exists a correlation, the global cannot be used as the fitness distribution of a specific node . This would distroy the structure of Eq (3) and (4), and consequently affect the existence of the solution. Hence, whether there exists a fitness access order so that the original condensation can be eliminated or suppressed, is an issue worthy of exploration. Intuitively, since the high-fitness nodes are more likely to evolve into the condensate nodes, the access to the network seems to prioritize the low fitness nodes, which may offer them more time to acquire larger degree to balance the high-fitness nodes. In other words, it is possible to utilize the first-mover advantage to counteract the fitness advantage. In the practical circumstances, such strategies do exist, such as preferential support policies for small or medium-sized enterprises, or the establishment of qualification for entry through the issuance of licenses in certain industries [26, 27]. Nonetheless, there has been a lack of a solid theoretical foundation of the rationality and effectiveness of the specific entry order.

C. Implementation of ordered access and methodology of determining the condensation transition in simulation

3.1. Fitness reordering algorithm

For a sequence of fitness elements, we design a reordering algorithm to adjust their arrangement and to achieve the ordered access processes. Taking the ascending reordering as an example, the steps are as follows:

(a) Arrange the sequence in ascending order based on their fitness values.

(b) Conduct rounds of active exchanges on the elements in the sequence: in round , if the element at the position has not previously undergone an active exchange, it selects an element in position according to the probability and actively exchanges the position with it, where represents the present distance between the two elements in the sequence. If the element has previously implemented an active exchange, it remains its position in the current round (in other words, each element can only undergo an active exchange at most once, but it can still be selected and be exchanged in other rounds).

Figure 2. illustrates the active exchange procedure in the reordering algorithm, where represents three positions in the sequence and different shapes represent the elements with different fitness. The red and green color indicates that the element has and has not undergone an active exchange respectively. The yellow arrows indicate the current position in the procedure. In Figure 2.(a), the element at position implements an active exchange with the element at position (blue directed solid line). At round (Figure 2.(b)), the element at position is exactly the one at in Figure 2.(a). Since it has already implemented an active exchange, no action will be taken in this round. But note that in the subsequent rounds, it can still be selected by other elements and be exchanged, as shown in Figure 2.(c), where the blue directed dashed lines illustrate the two objects that can be actively exchanged by the element at position . In fact, the algorithm is equivalent to constructing a spatial small-world network for the one-dimensional lattice composed of the fitness elements [28-29], where corresponds to the probability of supplementing directed long edges and the active exchange is equivalent to swapping the position of an element with its target pointed by the out-going link. The reason that the number of the active exchange of an element is limited to unity is to minimize the probability of an element being repeatedly exchanged back and forth. Considering that the expected in-degree of each element is , the probability is insignificant. The variable in is called the reshuffling index. When , the reshuffled fitness sequence returns to the random permutation as the original model and when , the fitness sequence remains its initial ascending order. Therefore, adjusts the order of the fitness arrangement, which can be taken as the intervention degree of the regulation to the original free market.

We can change step (a) into the rearrangement in descending order and implement a descending reshuffling procedure according to step (b), where the reshuffling index is defined as a negative value. This symmetric stipulation allows us to achieve unbiased reshufflings and allow us to study the condensation transition throughout the entire space of . It is noted that the reshuffling procedure cannot include all possible permutations. Nonetheless the power-law ensure that the generated long edges are of various scales, which can cover all kinds of arrangements.

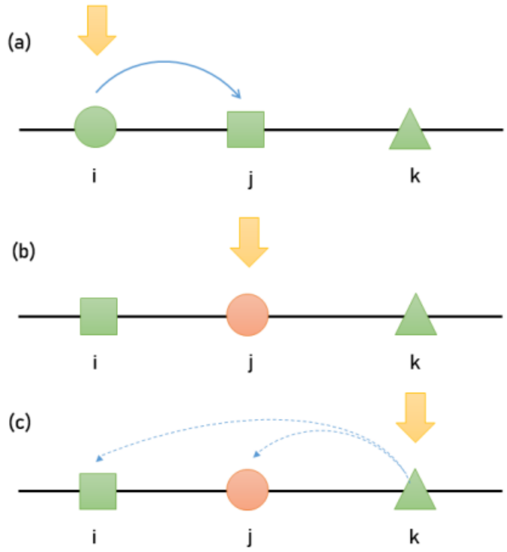

Figure 2. Schematic diagram of active exchange procedure in fitness rearrangement algorithm.

3.2. Methodology of Determining the condensation transition

The temporal nature of the fitness access makes the analytical derivation difficult. Therefore, we turn to the simulations and the numerical analysis of the condensation characteristics. In principle, the occurrence of the condensation can be determined by tracking the ratio of the maximum degree to the evolution time . However, when the system approaches the critical point, often decays slowly, making it difficult to discern its true trend and causing inevitable subjectivity and uncertainty in the determination of the critical point. To apply a more reliable method, we use the finite-size scaling analysis proposed in reference [7], which studies the relation between and the index as increases, where is the maximum degree at the end of the evolution of a simulated network of size and represents the average over multiple simulations. If the condensation occurs, is independent of so that the data points at the same is expected to overlap. Otherwise the data points are separated. Therefore, the point at which the data points begin to separate corresponds to the critical for the condensation. To provide a definite criterion for this critical point, we introduce the sensitivity coefficient , defined as

$$\chi = \sqrt{\langle k_{max}^2 / N^2 \rangle - \langle k_{max} / N \rangle^2} \quad (5)$$

The critical can be determined by the maximum of since significant fluctuations are supposed to occur at the transition point.

D. Case study and discussion

We use the fitness distribution suggested in reference [7] in our study:

$$\rho(\eta) = -\frac{2}{\beta^2}\frac{\ln \eta}{\eta} \quad (6)$$

where $\eta \in [e^{-\beta}, 1]$ $\beta > 1.3$. When , the fitness model is in a condensed state, whose degree increases with β [7]. For an actual economic system, β can be considered as a measure of the deterioration degree of the market environment.

We generate N fitness elements according to Eq (6) and sort them by using the reshuffling algorithm. The sorted elements are then sequentially added to the network for simulation. Fig. 3 illustrates the temporal evolution of k_{max}/t for different values of . The simulation parameters are set as $\beta = 3$, $m = 1$ and k_{max}/t . When , the evolution is equivalent to the original fitness model. The evolves to be time independent, indicating the emergence of the condensation as expected. When $\alpha = 0$, the regulation strategy introduces an positive correlation between fitness and evolution time. Yet, the correlation is weak and does not alter the condensation state. When the correlation is strengthened to a certain degree (e.g.,), exhibits a clear power-law decay, indicating the regulation manage to inhibit the condensation. Remarkably, when $\alpha = 0$ also shows a slow decay, suggesting that the condensation can still be inhibited when fitness is arranged in descending order.

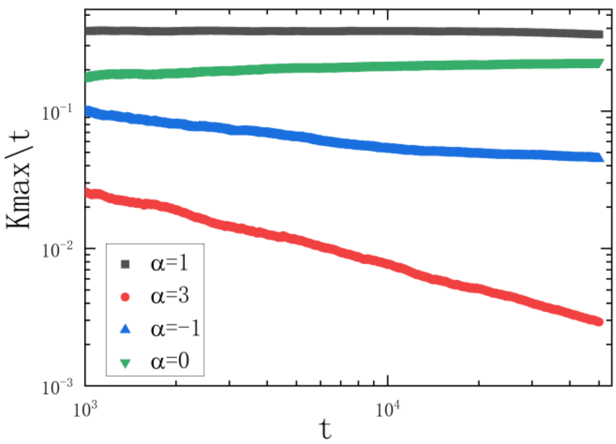

Figure 3. Evolution of the proportion of network maximum degree under different rearrangement indices when .

In order to measure the critical index α_c accurately, we investigate the relation $\langle k_{\max}/N \rangle$ vs α for three different network sizes $N = 10^3, 10^4, 10^5$. Fig. 4(a) and 5(a) present the corresponding results for $\beta = 2$ and $\beta = 3$, where each data point represents the average of 30 independent simulations. The results show that in both figures there exist a region across where the data points overlap, which suggests that the network is still in a condensed state. However, as gets far away from zero to a certain extent, the data points of different network sizes gradually separate and then the ratio $\langle k_{\max}/N \rangle$ displays a decrease with N , indicating that the regulation strategy has inhibited the condensation. The two splits of the data points, as seen in both $\alpha > 0$ and $\alpha < 0$ regions, correspond to the two critical indices α_c^+ and α_c^- for ascending and descending reorganization respectively, which are consistent with the peaks of the sensitivity coefficient , as shown in Fig. 4(b) and 5(b). We have $\alpha_c^+ = 1$ and $\alpha_c^- = -0.2$ for $\beta = 2$, and $\alpha_c^+ = 1.5$ and $\alpha_c^- = -0.4$ for $\beta = 3$.

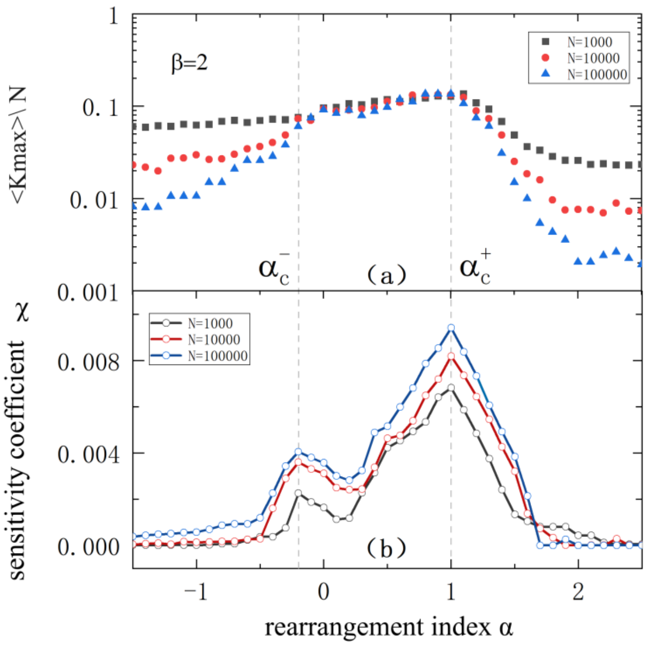

Figure 4. Relationship between the proportion of the average maximum degree at the final state (a), sensitivity coefficient (b) and rearrangement index when $\beta = 2$.

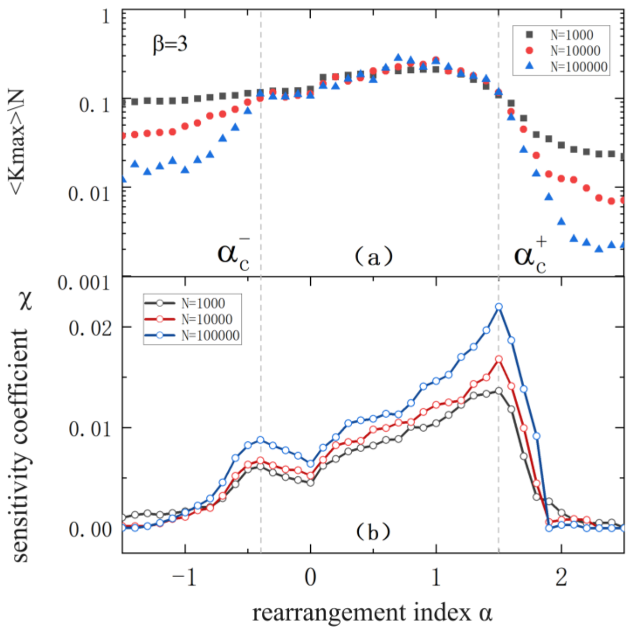

Figure 5. Relationship between the proportion of the final state average maximum degree (a), sensitivity coefficient (b) and rearrangement index when $\beta = 3$.

Based on the aforementioned method, we conducted the measurements of multiple and for different and eventually arrive at a condensation phase diagram. As depicted in Fig. 6, the two curves formed by (red square) and (blue circle) delineate the boundaries of the regulation effectiveness and describe the parameter regime of the condensed and non-condensed states, which leads to the following conclusions:

(1) For a specific condensation strength , there exists a corresponding range of the rearrangement indices, within which condensation occurs while outside which, the condensation is effectively suppressed by the regulation strategy.

(2) The range of rearrangement indices spans the zero point, indicating that there can be a certain access control strategy for both ascending and descending rearrangements to achieve condensation suppression.

(3) The critical rearrangement indices of both ascending and descending rearrangements exhibit positive correlations with the condensation strength . The range of the indices increases with , indicating more intervention needed to achieve effective regulation in a more deteriorated environment.

(4) There is an apparent difference between the ascending and descending rearrangements on their correlation pattern of the critical rearrangement index and the condensation strength . In the case of ascending rearrangements, the critical rearrangement index increases slowly with , while for descending rearrangements it decreases in a remarkably nonlinear manner, showing a more rapid divergent trend.

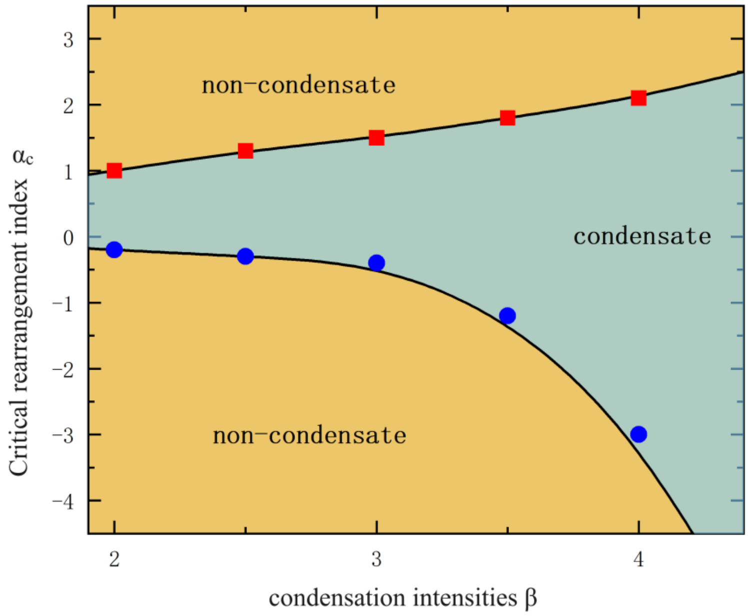

Figure 6. Critical rearrangement index under different condensation intensities, and the corresponding condensed phase diagram.

From the perspective of the market management, conclusion (1) gives a theoretical basis for the validity of the ordered access strategies in suppressing the monopolies, while conclusion (3) illustrates how the market intervention is affected by the market environment. Specifically, for a market with a certain degree of deterioration, how much intervention we should introduce to the access order to eliminate the monopolies. Conclusion (2) indicates that regulation possesses a considerable degree of freedom, which implies that, there leaves margins for designing extra rules to accommodate other needs and objectives while eliminating monopolies.

A noteworthy issue is why descending reordering can also achieve suppression of the condensation. If a high fitness node gains a first-mover advantage, it can rapidly acquire connections and evolve into a condensed node. To avoid the consequences, it is essential to introduce more high-fitness nodes promptly to balance the competitiveness. While this approach may not lead to a more equitable allocation of resources, it can indeed prevent these nodes from dominating the network. The specific descending reordering index in our model can just achieve this effect and suppress the condensation. However, the effectiveness of this approach is subject to the fitness distribution. If the parameter is every large, the tail of the fitness distribution will be so uneven that the largest fitness in the network is far greater than other ones, then there is not any single node that is powerful enough to compete against it. In the context of the ascending reordering, it is still possible to reach an equilibrium of the competitiveness between the entire network nodes and the largest-fitness node by delaying the its entry. However in the case of the descending reordering, the loss of the checks and balances results in that any measure that prioritizes the entry of the maximum fitness will promote the condensation, which implies that the regulation of descending reordering has its inherent limit. As the parameter increases, the system continuously approaches this limit and inevitably leads to the rapid divergence of the reordering index, as indicated by conclusion (4).

E. Conclusion

We introduce a fitness ordered access strategy to the fitness model and investigate its effect on regulating the condensation of the network topology. By the extensive Monte Carlo simulations and the finite-size scaling analysis, we determine the critical reordering index of the regulation, and establish its relation with the condensation strength, and eventually construct the phase diagram of the condensation. This phase diagram indicates the effectiveness of ordered access strategy in suppressing the monopolies, and quantitatively characterizes the intervention level required for a particular market deterioration as well as the establishment of entry rules to curb the monopolies.

We observe an accelerated divergence effect of the critical reordering index with the condensate strength under the reversed reordering, which is of special significance for understanding the contemporary internet- platform monopolies. The formation and evolution of internet platforms generally involves the large-scale capital investment, the user growth and high financing, which can generate multi-level monopolistic forces. Firstly, during the initial stage of the financial investment, the capital groups, relying on their capital and information advantages, usually have the lower entry barriers so that they can access the market earlier. Then due to their scale advantage, they can raise the barriers and hinder the entry of other small and medium-sized capital. This implies that the internet platforms inherently possess the reverse reordering attribute of our model. Moreover, instead of the competition, the large capital entities may engage in strong cooperation through financing, leading to the rapid formation of super-fitness entities in short term. The situation associated with the above two arguments corresponds to the divergence area of our model with high condensation intensity and a large reverse reordering index. As the worst case, it will rapidly lead to the formation of the condensed nodes that are stable and difficult to eliminate. In other words, from the perspective of complex network theory, the unreasonable entry order and the unrestricted financing scale are the key factors that are responsible for the monopolization of internet platforms. Based on the theoretical results of this paper, we have some suggestions for the Antitrust policy: (1) Establish a reasonable market regulation to curb the inherent reverse reordering attribute of internet platforms, which may involve encouraging and supporting the access of small and medium-sized enterprises to the relevant markets and breaking down the high access barriers developed by large capital entities. (2) Limit the financing scale and prohibit the malicious acquisition behaviors to avoid the formation of super-fitness nodes.(3) Establish a sound market mechanism to promote internal competition among the enterprises.

In comparison to the fitness adjustment scheme proposed in reference [25], our ordered access strategy only requires the assessment of the fitness of individuals who intends to enter the market, and thereby significantly reduces the resource and time costs. In fact, similar policies have been implemented in some industries [26,27], and our study theoretically reveal the relation between access rules and market monopolization from the perspective of complex networks. Although the practical access scheme still requires specialized guidance and improvement, based on the validity of the fitness models in describing the evolution of economic systems and their monopolistic mechanisms as confirmed by the current research, we believe that the conclusions of this paper are not sensitive to the specific parameters of the system, but rather can be applied to a wide range of socio-economic fields.

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